SIMULTANEOUS AERODYNAMIC ANALYSIS AND DESIGN OPTIMIZATION (SAADO) FOR A 3-D RIGID WING

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Abstract

The formulation and implementation of an optimization method called Simultaneous Aerodynamic Analysis and Design Optimization (SAADO) is presented and applied to a simple 3D wing problem. The method aims to reduce the computational expense incurred in performing shape optimization using state-of-the-art CFD flow analysis and sensitivity analysis tools. Results for this small problem show that the method reaches the same local optimum as conventional optimization methods and does so in about half the computational time.

Nomenclature

b	wing semispan
C_D	drag coefficient
C_l	rolling moment coefficient
C_L	lift coefficient
C_M	pitching moment coefficient
C_p	pressure coefficient
c_r	wing root chord
c_t	wing tip chord
F	objective function
g	constraints
M_∞	free-stream Mach number
Q	flow field variables (state variables) at each CFD mesh point
ΔQ_1	change in flow solver field variables due to better convergence
ΔQ_2	change in flow solver field variables due to design changes
R	residual field of the state equations at each CFD mesh point
$\lvert R/R_0 \rvert$	norm of the residual normalized by its initial value
S	semispan wing planform area
T	twist angle at wing tip, positive for leading edge up
X	CFD volume mesh
X_{LE}	vector location of wing root leading edge
x/c	chordwise location normalized by local wing section chord
x_t	longitudinal location of wing tip trailing edge
z_r	root section camber
α	free-stream angle-of-attack
β	design variables

Δ operator which indicates a change in a variable

δ size of fictitious side bound on design variables

γ line search parameter

superscripts:

$*$ designates updated value

$'$ gradient with respect to design variables

Introduction

Simultaneous Aerodynamic Analysis and Design Optimization (SAADO) is a procedure which incorporates design improvement within the iteratively solved (nonlinear) aerodynamic analysis so as to achieve fully converged flow solutions only near an optimal design. Overall computational efficiency is achieved because the many expensive iterative (nonlinear) solutions for non-optimal design parameters are not converged (i.e., obtained) at each optimization step. One obtains the design in the equivalent of a few (rather than many) multiples of the computational time for a single, fully converged flow analysis. SAADO and similar procedures for simultaneous analysis and design (SAND) developed by others are noted and discussed by Newman et al.[1] These SAND procedures appear to be best suited for applications where the discipline analyses involved in the design are nonlinear and solved iteratively. Generally, convergence of these discipline analyses (i.e., state equations) is viewed as an equality constraint in an optimization problem. From this latter point of view, the SAADO method proceeds through infeasible regions of the (β, Q) design space. A further advantage of SAADO is the efficient utilization of existing discipline analysis codes (without internal changes), augmented with sensitivity or gradient information, and yet effectively coupled more tightly than is done in conventional gradient-based optimization procedures, referred to as nested analysis and design (NAND) procedures.[1] A recent overview of aerodynamic shape optimization[2] discusses both NAND and SAND procedures in the context of current steady aerodynamic optimization research.

For single-discipline design problems, the distinction between NAND and SAND procedures is fairly clear and readily seen. With respect to discipline feasibility (i.e., convergence of the iteratively (generally nonlinear) solved state equations), these procedures can be viewed as accomplishing design by using only very well converged discipline solutions (NAND) or as converging a sequence of discipline solutions from poorly to well as the design progresses (SAND). However, the problem formulation and solution algorithms may differ considerably. About twenty SAND references are quoted by Newman et al.[1] and Newman et al.[2]; these references discuss a variety of formulations, algorithms, and results for single-discipline problems (mostly CFD applications) in the sense of SAND defined above. For multidisciplinary design optimization problems, the distinction between NAND and SAND is somewhat blurred because there are feasibility considerations with respect to all the individual discipline state equations as well as with respect to the multidisciplinary system compatibility and constraints. A number of the papers in Ref. 3 discuss MDO formulations and algorithms that are called SAND-like. However, not all of these latter MDO procedures appear to agree with the sense of SAND defined above and used herein; one that does is Ref.4.

The computational feasibility of SAADO for quasi 1-D nozzle shape design based on the Euler equation CFD approximation was demonstrated by Hou et al.[5] and Mani.[6] Application of SAADO for turbulent transonic airfoil shape design based on a 2-D thin-layer Navier-Stokes CFD approximation was demonstrated and reported in a later paper by Hou et al.[7] Both of these application results are summarized and briefly discussed in Ref. 1. These SAADO procedures utilized quasi-analytical sensitivity derivatives which were obtained from hand-differentiated code for the initial quasi 1-D application[5, 6] and from automatically differentiated code for both the 2-D airfoil application[7] and the present 3-D wing application. Different optimization techniques have also been used in these SAADO procedures. Our initial 3-D wing results from SAADO, obtained with several optimization techniques, are given in this paper. The problem was a simple wing planform design where changes in design variables were sought to produce improvement in the lift-to-drag ratio subject to lift, pitching moment, and rolling moment (i.e., solution dependent) constraints, as well as geometric constraints. Our ultimate goal is simultaneous aerodynamic and structural wing design.

Problem Description

To evaluate the efficacy of this SAADO procedure in 3-D, it is applied herein to a simple rigid wing alone. The wing consisted of a trapezoidal planform with a rounded tip. It was parameterized by fifteen variables; five described the planform, and five each described the root and tip section shapes. A schematic of the wing and its associated planform parameters is shown in Fig. 1. The baseline wing section varied linearly from an NACA 0012 at the root to an NACA 0008 at the tip. The specific parameters selected as design variables in the sample optimization problems are identified in the section entitled Results. The objective function to be minimized was the negative of the lift-to-drag ratio, –L/D. In lieu of additional discipline interactions, both aerodynamic and geometric constraints were imposed.

The aerodynamic constraints were:

– lower limit on lift, $C_L * S$, in lieu of a minimum payload requirement
– upper limit on rolling moment, C_l, in lieu of a maximum bending moment
– upper limit on pitching moment, C_M, in lieu of a trim constraint

The purely geometric constraints were:

– minimum leading edge radius, in lieu of a manufacturing requirement
– minimum wing section thickness, in lieu of a structural interaction
– side constraints (bounds) on the active design variables

SAADO Procedure

The SAADO approach formulates the design-optimization problem as follows:

$$\min_{\beta,Q} F(Q, X(\beta), \beta) \tag{1}$$

subject to

$$g_i(Q, X(\beta), \beta) \le 0; \qquad i = 1,2,\ldots,m \tag{2}$$

and

$$R(Q, X(\beta), \beta) = 0\ . \tag{3}$$

Recall that Q, R, and X are very large vectors. This formulation treats the state variables, Q, as part of the set of independent design variables, and considers the state equations to be constraints. Because satisfaction of the equality constraints of Eq. (3) is required only at the final optimum solution, the steady-state aerodynamic field equations are not converged at every design-optimization iteration. The easing of that restriction can significantly reduce the excessively large computational cost incurred in the conventional approach. However, this advantage would likely be offset by the very large increase in the number of design variables and equality constraint functions, unless some remedial procedure is adopted.

The SAADO method begins with a linearized design-optimization problem which is solved for the most favorable change in the design variables, $\Delta\beta$, as well as for the changes in the state variables, ΔQ; that is,

$$\begin{aligned}&\min_{\Delta\beta,\Delta Q} F(Q, X, \beta)\\ &+\frac{\partial F}{\partial Q}\Delta Q + \left(\frac{\partial F}{\partial X}X' + \frac{\partial F}{\partial \beta}\right)\Delta\beta\end{aligned} \tag{4}$$

subject to

$$\begin{aligned}&g_i(Q, X, \beta) + \frac{\partial g_i}{\partial Q}\Delta Q\\ &+\left(\frac{\partial g_i}{\partial X}X' + \frac{\partial g_i}{\partial \beta}\right)\Delta\beta \le 0; \qquad i = 1,2,\ldots,m\end{aligned} \tag{5}$$

and

$$\begin{aligned}&R(Q, X, \beta) + \frac{\partial R}{\partial Q}\Delta Q\\ &+\left(\frac{\partial R}{\partial X}X' + \frac{\partial R}{\partial \beta}\right)\Delta\beta = 0\end{aligned} \tag{6}$$

where Eqs. (4), (5) and (6) are linearized approximations of Eqs. (1), (2) and (3), respectively. In this formulation, the residual of the non-linear aerodynamic field equations, $R(Q,X,\beta)$, is not required to be zero (reach target) until the final optimum design is achieved. The linearized problem of Eqs. (4), (5), and (6) is difficult to solve directly because of the number of design variables and equality constraint equations. This difficulty is overcome for the direct differentiation method by using direct substitution to remove ΔQ and Eq. (6) altogether from this linearized problem. The discrete adjoint method described in the Appendix is another way to overcome the difficulty. However, the remainder of this paper describes only the use of the direct differentiation method for formulating the SAADO procedure and its application to optimization problems.

Equation (6) provides a linear relation to represent ΔQ in terms of $\Delta\beta$ as

$$\Delta Q = \Delta Q_1 + \Delta Q_2 \Delta\beta \tag{7}$$

where vector ΔQ_1 and matrix ΔQ_2 are solutions of the following equations

$$\frac{\partial R}{\partial Q}\Delta Q_1 = -R \tag{8}$$

$$\frac{\partial R}{\partial Q}\Delta Q_2 = -\left(\frac{\partial R}{\partial X}X' + \frac{\partial R}{\partial \beta}\right). \tag{9}$$

Note that the number of columns of matrix ΔQ_2 is equal to the number of design variables, β. A new linearized problem with $\Delta\beta$ as the only design variables can be obtained by substituting Eq (7) into Eqs. (4) and (5) for ΔQ:

$$\begin{aligned} &\min_{\Delta\beta} F(Q, X, \beta) + \frac{\partial F}{\partial Q}\Delta Q_1 \\ &+ \left(\frac{\partial F}{\partial Q}\Delta Q_2 + \frac{\partial F}{\partial X}X' + \frac{\partial F}{\partial \beta} \right)\Delta\beta \end{aligned} \qquad (10)$$

subject to

$$\begin{aligned} &g_i(Q, X, \beta) + \frac{\partial g_i}{\partial Q}\Delta Q_1 \\ &+ \left(\frac{\partial g_i}{\partial Q}\Delta Q_2 + \frac{\partial g_i}{\partial X}X' + \frac{\partial g_i}{\partial \beta} \right)\Delta\beta \le 0; \\ &\quad i = 1,2,\ldots, m \end{aligned} \qquad (11)$$

Once established, this linearized problem can be solved using any mathematical programming technique for design changes, $\Delta\beta$. The associated changes in ΔQ can be calculated using Eq (6) or equivalently, Eqs. (7), (8), and (9). A one-dimensional search on the step size parameter γ is then performed in order to find the updated values, $\beta^*=\beta+\gamma\Delta\beta$, the new mesh, $X^*=X(\beta^*)$, and $Q^*=Q+\Delta Q^*$ where the search procedure must solve a nonlinear optimization problem of the form

$$\min_{\gamma} F(Q^*, X^*, \beta^*) \qquad (12)$$

subject to

$$g_i(Q^*, X^*, \beta^*) \le 0; \qquad i = 1,2,\ldots, m \qquad (13)$$

and

$$R(Q^*, X^*, \beta^*) = 0 \qquad (14)$$

The step size γ is the only design variable. Again it is noted for emphasis that the equality constraints, Eq. (14), are not required to be zero (reach target) until the final optimum design; violations of these equality constraints should simply be progressively reduced during the SAADO procedure. Therefore, the updated Q^* in this study is defined as $Q^*=Q+\Delta Q^*$, which satisfies the first order approximations Eq. (14) as

$$\begin{aligned} &R(Q, X, \beta) + \frac{\partial R}{\partial Q}\Delta Q^* \\ &+ \left(\frac{\partial R}{\partial X}X' + \frac{\partial R}{\partial \beta} \right)(\gamma\Delta\beta) = 0 \end{aligned} \qquad (15)$$

Equation (15) may be solved using Eqs (8) and (9), which give the updated Q^* as $Q^*=Q+\Delta Q_1+\gamma\Delta Q_2\Delta\beta$. Therefore, the update of Q includes two parts: ΔQ_1 and ΔQ_2. The former is due to better convergence of the flow solver, whereas the latter is due to changes in the design variables. In fact, ΔQ_2 approaches the flow field sensitivities, Q', as the solution becomes better converged. The appearance of ΔQ_1 in the formulation makes the SAADO approach different from the conventional NAND aerodynamic optimization method. The ΔQ_1 not only constitutes a change in Q but also plays an important role in defining the constraint violation of Eq. (11). Since ΔQ_1, as shown in Eq. (8), represents a single Newton's iteration on the flow equations, it is possible to approximate it as the change in Q as a result of several Newton's iterations to improve the quality of the flow solution. A schematic of the present SAADO procedure is shown in Fig. 2.

Computational Tools and Models

The aerodynamic flow analysis code used for this study is a version of the CFL3D code[8]. Only Euler analyses are performed for this work, although the code is capable of solving the Navier-Stokes equations with any of several turbulence models. The derivative version of this code, which was used for aerodynamic sensitivity analysis, was generated by an unconventional application[9] of the automatic differentiation code ADIFOR[10, 11] to produce a relatively efficient, direct mode, gradient analysis code, CFL3D.ADII.[12] It should be pointed out that the ADIFOR process produces a discretized derivative code that is consistent with the discretized function analysis code. The addition of a stopping criterion based on the norm of the residual of the field equations was the only modification of the CFL3D.ADII code made to accommodate the SAADO procedure.

The surface geometry was generated based on the parameters described in the previous section by a code utilizing the Rapid Aircraft Parameterization Input Design (RAPID) technique developed by Smith, et al.[13] This code was preprocessed with ADIFOR to generate a code capable of producing sensitivity derivatives as well.

The CFD volume mesh needed by the flow analysis code was generated using a version of the CSCMDO[14] grid generation code. The associated grid sensitivity derivatives needed by the flow sensitivity analysis were generated with an automatically differentiated version of CSCMDO.[15] In addition to the parameterized surface mesh and accompanying gradients, CSCMDO requires a baseline volume mesh of similar shape and identical topology. The 40,000 grid point baseline volume mesh of C-O topology used in the present examples was obtained with the WTCO code.[16] This mesh is admittedly particularly coarse by current CFD analysis standards.

Several optimization techniques were used in this study. Both conventional and SAADO procedures were implemented, run, and compared. The Sequential Quadratic Programming method and the Method of Feasible Directions as implemented in the DOT[17] optimization software were used and the results were designated SQP and MFD, respectively, when run in the conventional (NAND) mode. The same optimization methods were also used in a SAADO or SAND mode (that is, partially converged CFD analysis and sensitivity results at each iteration) and these results were designated SSQP and SMFD. In addition, a rather inefficient, unsophisticated optimization technique was devised for use in the SAADO mode in order to provide dynamic direct access to and control of parameters typically internally controlled by packaged optimizers. This technique is a sequential linear method consisting of

– formulation of a merit function as weighted sum of objective function and active constraints
– determination of the step direction by steepest descent of the merit function
– determination of the step size using a one-dimensional line search

Results are designated SDMF when obtained in the conventional mode and SSDMF when obtained in the SAADO mode.

All procedures were executed with an SGI OCTANE™ workstation with a 250Mhz R10000™ processor. The 785MB RAM was sufficient for the largest code, CFL3D.ADII, to fit in core for the 40,000-point mesh and number of design variables considered herein.

Results

Most of the results shown in this work are for a design problem involving only two design variables out of the fifteen available wing parameters. Preliminary results for a five-design-variable problem are briefly discussed in the subsequent subsection. In the last subsection, these present SAADO results are discussed in the context of other SAND approaches.

Two-Design-Variable Problems

Many of the benefits and many of the problems associated with SAADO are demonstrated with just two design variables. A benefit of studying just two design variables is the facility to visualize the optimization process. In the course of another approximation and optimization study[18], databases of computational results were created for each of several grids for the present objective function as a function of c_t and x_t with the same CFD code, wing, and flow conditions as used herein. One of those grids was identical to that used in this study. The results in this database were converged to $|R/R_0| = 10^{-6}$. Thus, contour plots of the present objective function are available; on such plots, we can show the paths taken through design space by the several optimization techniques.

Subsonic Problem-The flow conditions for the subsonic flow case were $M_\infty = 0.5$ and $\alpha = 3°$. Figure 3a shows the objective function contour plot for this case; the two SAADO mode results are those labeled SSDMF and SSQP whereas the conventional mode result is that labeled SQP. The rolling moment constraint boundary with its corresponding infeasible region appears on this plot; however, the other constraints are feasible throughout the domain shown. This (c_t-x_t) domain consists of that region bounded by the design variable side constraints. All three optimization results lie very close to the minimum.

Figure 3b shows the different paths taken by these three optimization techniques. Because none of these optimization paths approach the constraint boundary, none of the constraints are active for these subsonic case results. All of these paths start at (1,1) in the (c_t, x_t) design space. The domain has been truncated for these plots. The paths taken by the SQP and SSQP methods appear to be much more efficient in terms of the total number of calls to the analysis code. Even with analysis and gradients not as well converged, the step direction and size determined by the SSQP are quite similar to those of the SQP method. The SSDMF method takes smaller steps on a distinctly more circuitous path. This circuitous path is partially due to less effective use of past iteration results and also due to a somewhat simplistic method of determining the step size and direction. Definitive computational time comparisons for these subsonic cases are not available due to omissions in the code instrumentation. The results are presented to show that for two optimization techniques, the SAADO (SAND) method finds the same optimum as the conventional (NAND) method. Further, these results suggest that a more sophisticated optimization procedure could substantially improve the SSDMF result, while greater user control of the step size, particularly early in the procedure, could further decrease the computational cost for the SSQP result.

Figure 4 shows the wing planform and the surface pressure coefficient results for the subsonic cases. The three "optimized" planform shapes are very similar and can be seen to result in essentially the same chordwise pressure distributions. Furthermore, all of these distributions differ little from the original, even where the local chords differ, away from the wing root.

Supercritical Problem- Most of the experimentation with SAADO was conducted for a problem which is more interesting both from the point of view of the aerodynamic analysis and of the optimization problem. The flow conditions for the supercritical flow case were $M_\infty = 0.8$ and $\alpha = 1°$. A shock was present over a substantial spanwise portion of the wing and there were active constraints. Figure 5a shows the objective function contour plot for the two design variable problem. The shaded region denotes the infeasible region of the lift constraint. The other constraints were not active. The final iterates for eight optimization technique results are shown to lie near the optimal point of the feasible region. These eight techniques consist of three conventional NAND optimizations (SQP, MFD and SDMF) and their corresponding SAND modes (SSQP, SMFD and SSDMF) as discussed in a previous section, as well as two modifications to the SSQP, denoted as SSQP, δ=0.10 and SSQP, δ=0.15. The parameter δ simply provides a means to limit the change in design variables at each optimization step by dynamically adjusting the side constraints. Further discussion concerning results from these latter two techniques appears in the subsequent subsection.

Figure 5b shows the different paths taken by these eight optimization techniques. All of these optimization paths approach and cross the lift constraint boundary. Thus, this constraint was active and, at times, violated in all of the techniques for this supercritical problem. As in the subsonic problem, all of these paths start at (1,1) in the (c_t, x_t) design space, and the domain has been truncated for these plots. Here too, the less-converged analysis and gradient data used by the SSQP method do not seem to impair its ability to determine the step direction and size. Again, the results are similar to those of the conventional SQP method. The simplistic SDMF and SSDMF methods still take smaller steps on less direct paths, and once more, these circuitous paths are due to ineffective use of information from past optimization iteration results and a simplistic means of determining the step size and direction. The step size for the SDMF decreased substantially when the constraint was encountered. The SDMF required a few more optimization steps than the SSDMF to reach the minimum. As expected, the MFD spends less time in the infeasible region than the other optimization techniques. However, the SMFD does not appear to track the MFD optimization path very well.

Figure 6 shows the wing planform and the surface pressure coefficient results for the supercritical cases. Since all of the "optimized" planform shapes are essentially the same and result in the same chordwise pressure distribution, results from only three optimization techniques are shown. The shock wave on the original wing has been weakened substantially in the optimized cases, as would be expected.

Computation Cost Comparisons- In view of the consistency of the NAND and SAND optimization results, the measure of success or failure of the SAADO procedure is then its relative computational expense. Figure 7 shows the relative cost, based on accumulated CPU time, for the procedures using the various methods. The cost of the geometry generator, mesh generator and optimization driver are not included because their contributions are minimal relative to the cost of the flow solver and flow sensitivity solver. The cost shown is for flow analyses and gradient analyses and is normalized by the cost of one full analysis to the target residual. The SAADO method primarily reduces the cost of the flow analysis. In this regard, the SAADO method does show improvement over its conventional counterpart for all methods.

The SAADO method appears to make the most overall improvement for the Method of Feasible Directions (MFD vs. SMFD) and the Steepest Descent Method with the Merit Function (SDMF vs. SSDMF). The least relative reduction is shown for the Sequential Quadratic Programming method (SQP vs. SSQP). Note that for the conventional methods, the MFD is dominated by flow analysis whereas the SDMF is dominated by sensitivity analysis; the SQP is relatively balanced. On the computer platform used in this study, the ADIFOR-generated gradient analysis is relatively inefficient compared to the flow analysis.

For the SAND methods, the smaller steps taken in the SSDMF path relative to those in the SSQP path, result in smaller perturbations in the flow and gradient fields. In addition, for each step in the SSDMF, the convergence level for the subsequent step is not as stringent as that required in the SSQP. Compared to the number of flow solver iterations required at each design step in SSQP and conventional SQP, relatively few are required in the SSDMF. However, this savings is traded off with the relatively large number of design steps required for SSDMF. Some combination of the two methods, such that small steps are taken along a more direct path, may lead to improved efficiency. That improved efficiency was the idea behind the two SSQP methods labeled δ= 0.1 and δ=0.15. The side constraints were adjusted at each call to DOT so as to limit the step size. However, an overall detrimental effect on the method resulted, since each time this fictitious side constraint was encountered, DOT called for recalculation of the gradients.

The most computational cost for any of these methods, other than MFD and SMFD, is spent computing gradients, even though none of the gradient residual ratios were converged below three orders of magnitude. Early in the respective processes, the gradients for SSDMF in particular and also the DOT methods were not well converged. As the number of design variables is increased, this proportion will grow nearly linearly. The need for faster gradient calculations is apparent. Hou et al.[1] estimated a considerable speed-up attributed to using hand-differentiated adjoint code for the 2-D Euler equations. Fortunately, automatic code generation tools for more easily and efficiently computing gradients using adjoint methods[19] are on the horizon.

Five-Design-Variable Problem

Limited preliminary results are given and briefly discussed in this section for the same wing shape optimization problem but with five design variables. These five design variables are root camber (z_r), tip chord (c_t), tip setback (x_t), wing semispan (b), and tip twist (T). Table 1 presents a comparison of the conventional SQP (NAND) and SSQP (SAND) results. It can be seen from the objective function and final design variable values that these two procedures did not arrive at exactly the same place in design space. However, both had the same two active constraints (rolling moment and pitching moment) and active side constraints (wing semispan and tip setback). The reduction in relative optimization cost, based on CPU time, for SAADO is comparable to that obtained in the two design variable cases and the total optimization cost scales with the number of design variables as it should for gradients obtained via direct differentiation.

Further Discussion

The relative cost, based on CPU timing ratios, for the SAADO (SAND) versus conventional (NAND) procedures applied to these present small 3-D aerodynamic shape design optimization problems range from about one-half to seven-tenths. This range is very similar to that reported for 2-D nonlinear aerodynamic shape design optimization in Refs. 1 and 4, even though many of the computational details differ. The results given in Ref. 1 were for a turbulent transonic flow with shock waves computed using a Navier-Stokes code; a direct differentiation approach (using ADIFOR) was used for the sensitivity analysis. The results reported in Ref. 4 were for a compressible flow without shock waves computed using a nonlinear potential flow code; an adjoint approach was used for the sensitivity analysis. Since these two optimization problems were also not the same, then, no timing comparison between these adjoint and direct differentiation solution approaches would be meaningful. As indicated earlier, an expected speed-up for using an adjoint, instead of the direct differentiation, approach was estimated in Ref. 1.

Ghattas and Bark[20] recently reported 2-D and 3-D results for optimal control of steady incompressible Navier-Stokes flow which demonstrate an order-of-magnitude reduction of CPU time for a SAND approach versus a NAND approach. These results were obtained using reduced Hessian SQP methods that avoid converging the flow equations at each optimization iteration. The relationship of these methods with respect to other optimization techniques is also discussed in Ref. 20.

Several other SAND-like methods for simultaneous analysis and design are summarized and discussed by Ta'asan [21]. These methods are called "One-Shot" and "Pseudo-Time" and have been applied to aerodynamic shape design problems at several fidelities of CFD approximation, as noted in Ref. 21. These techniques have obtained an aerodynamic design in the equivalent of several analysis CPU times for some sample problems.

Concluding Remarks

This study has introduced an implementation of the SAADO technique for a simple 3-D wing. Initial results indicate that SAADO

1. finds the same local minimum as a conventional technique for the two-design-variable cases and a similar local minimum for a preliminary five-design-variable case
2. is computationally more efficient than a conventional gradient-based optimization technique
3. requires few modifications to the analysis and sensitivity analysis codes involved.

Initial results have illustrated the need for an adjoint method for efficiently computing the sensitivity derivatives. Fortunately, such automated adjoint code generation tools are on the brink of being readily available. The SAADO method of closely coupling the analysis and optimizer iteration loops requires optimization software that does not usurp all control. That is, the SAADO technique needs to have some control over certain parameters during the optimization process; these parameters are typically considered 'optimizer' parameters and are only set by the user initially.

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Appendix - SAADO Procedure Using a Discrete Adjoint Approach

The SAADO approach formulates the design-optimization problem as follows:

$$\min_{\beta,Q} F(Q, X(\beta), \beta) \qquad \text{(A1)}$$

subject to

$$g_i(Q, X(\beta), \beta) \le 0; \qquad i = 1,2,\ldots,m \qquad \text{(A2)}$$

and

$$R(Q, X(\beta), \beta) = 0\,. \qquad \text{(A3)}$$

Recall that Q, R, and X are very large vectors. This formulation treats the state variables, Q, as part of the set of independent design variables, and considers the state equations to be constraints. Because satisfaction of the equality constraints of Eq. (A3) is required only at the final optimum solution, the steady-state aerodynamic field equations are not converged at every design-optimization iteration. The easing of that restriction can significantly reduce the excessively large computational cost incurred in the conventional approach. However, this advantage would likely be offset by the very large increase in the number of design variables and equality constraint functions, unless some remedial procedure is adopted.

The SAADO method begins with a linearized design-optimization problem which is solved for the most favorable change in the design variables, $\Delta\beta$, as well as for the changes in the state variables, ΔQ; that is,

$$\begin{aligned} \min_{\Delta\beta,\Delta Q} & F(Q, X, \beta) + \frac{\partial F}{\partial Q}\Delta Q \\ & + \left(\frac{\partial F}{\partial X} X' + \frac{\partial F}{\partial \beta}\right)\Delta\beta \end{aligned} \qquad \text{(A4)}$$

subject to

$$\begin{aligned} & g_i(Q, X, \beta) + \frac{\partial g_i}{\partial Q}\Delta Q \\ & + \left(\frac{\partial g_i}{\partial X} X' + \frac{\partial g_i}{\partial \beta}\right)\Delta\beta \le 0; \qquad i = 1,2,\ldots,m \end{aligned} \qquad \text{(A5)}$$

and

$$\begin{aligned} & R(Q, X, \beta) + \frac{\partial R}{\partial Q}\Delta Q \\ & + \left(\frac{\partial R}{\partial X} X' + \frac{\partial R}{\partial \beta}\right)\Delta\beta = 0 \end{aligned} \qquad \text{(A6)}$$

where Eqs. (A4), (A5) and (A6) are linearized approximations of Eqs. (A1), (A2) and (A3), respectively. In this formulation, the residual of the non-linear aerodynamic field equations, $R(Q,X,\beta)$, is not required to be zero (reach target) until the final optimum design is achieved. The linearized problem of Eqs. (A4), (A5), and (A6) is difficult to solve directly because of the number of design variables and equality constraint equations. This difficulty is overcome for the adjoint method by the introduction of the adjoint vectors, λ_0 and λ_i which are solutions of the following adjoint equations

$$\left(\frac{\partial R}{\partial Q}\right)^T \lambda_0 = -\frac{\partial F}{\partial Q} \qquad \text{(A7)}$$

$$\left(\frac{\partial R}{\partial Q}\right)^T \lambda_i = -\frac{\partial g_i}{\partial Q}; \quad i = 1, 2, ...m \ . \qquad \text{(A8)}$$

The linearized problem of Eqs. (A4) to (A6) can then be formulated as

$$\begin{aligned} &\min_{\Delta\beta} F(Q, X, \beta) + \lambda_0{}^T R \\ &+\left[\left(\frac{\partial F}{\partial X} + \lambda_0{}^T \frac{\partial R}{\partial X}\right) X' + \left(\frac{\partial F}{\partial \beta} + \lambda_0{}^T \frac{\partial R}{\partial \beta}\right)\right]\Delta\beta \end{aligned} \qquad \text{(A9)}$$

subject to

$$\begin{aligned} &g_i(Q, X, \beta) + \lambda_i{}^T R \\ &+\left[\left(\frac{\partial g_i}{\partial X} + \lambda_i{}^T \frac{\partial R}{\partial X}\right) X' + \left(\frac{\partial g_i}{\partial \beta} + \lambda_i{}^T \frac{\partial R}{\partial \beta}\right)\right]\Delta\beta \le 0; \\ &\quad i = 1,2,..., m \end{aligned} \qquad \text{(A10)}$$

Once established, this linearized problem can be solved using any mathematical programming technique for design changes, $\Delta\beta$. A one-dimensional search is then performed for the step size parameter γ in order to find the updated values, $\beta^*=\beta+\gamma\Delta\beta$, $X^*=X(\beta^*)$, and Q^* where the search procedure must solve a nonlinear optimization problem of the form

$$\min_{\gamma} F(Q^*, X^*, \beta^*) \qquad \text{(A11)}$$

subject to

$$g_i(Q^*, X^*, \beta^*) \le 0; \qquad i = 1,2,\ldots, m \qquad \text{(A12)}$$

and

$$R(Q^*, X^*, \beta^*) = 0 \ . \qquad \text{(A13)}$$

The step size, γ, is the only design variable. Again it is noted for emphasis that the equality constraints, Eq. (A13), are not required to be zero (reach target) until the final optimum design; violations of these equality constraints should simply be progressively reduced during the SAADO procedure. Therefore, the updated Q^* in this study is defined as $Q^*=Q+\Delta Q^*$, which satisfies the first order approximations Eq. (A13) as

$$\begin{aligned} &R(Q, X, \beta) + \frac{\partial R}{\partial Q}\Delta Q^* \\ &+\left(\frac{\partial R}{\partial X} X' + \frac{\partial R}{\partial \beta}\right)(\gamma\Delta\beta) = 0 \end{aligned} \qquad \text{(A14)}$$

The update for Q^* need not be determined from Eq. (A14), but rather can be taken from the results of solving Eq. (A13) in the process of determining γ.

The SAADO procedure presented here is similar to that procedure which Ghattas and Bark[20] call Quasi-Newton Sequential Quadratic Programming. However, that procedure as presented did not provide for state variable dependent constraints and it was directly embedded in a particular mathematical programming technique, SQP.

Table 1. Comparison of Preliminary SQP and SSQP Results for the Five−Design−Variable Problem

	Initial value	SQP value	SSQP value
Objective function	−8.4	−28.9	−27.6
Number of active constraints	——	2	2
Number of active bounds	——	2	2
c_t	0.5	0.259	0.411
x_t	0.	4.00**	4.00**
z_r, %chord	0.	2.32	2.18
b	3.	4.98**	4.98**
T, deg.	0.	−1.92	−1.38
Relative function analysis cost	1.*	12.1	3.4
Relative gradient analysis cost	3.3***	27.4	24.8
Relative optimization cost	——	39.5	28.2

* CPU time for an analysis solution converged to $|R/R_0| \le 10^{-6}$ is 3090 sec.

** Design variable side bound is active

*** Gradient solutions converged to $|R/R_0| \le 10^{-2}$

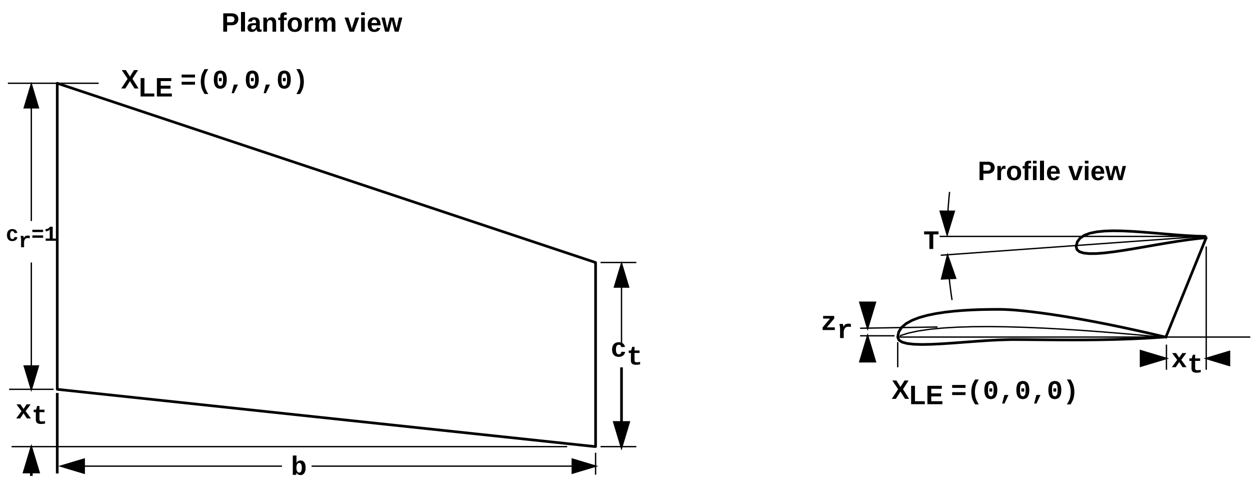

Fig. 1 Diagram of the parametric wing model.

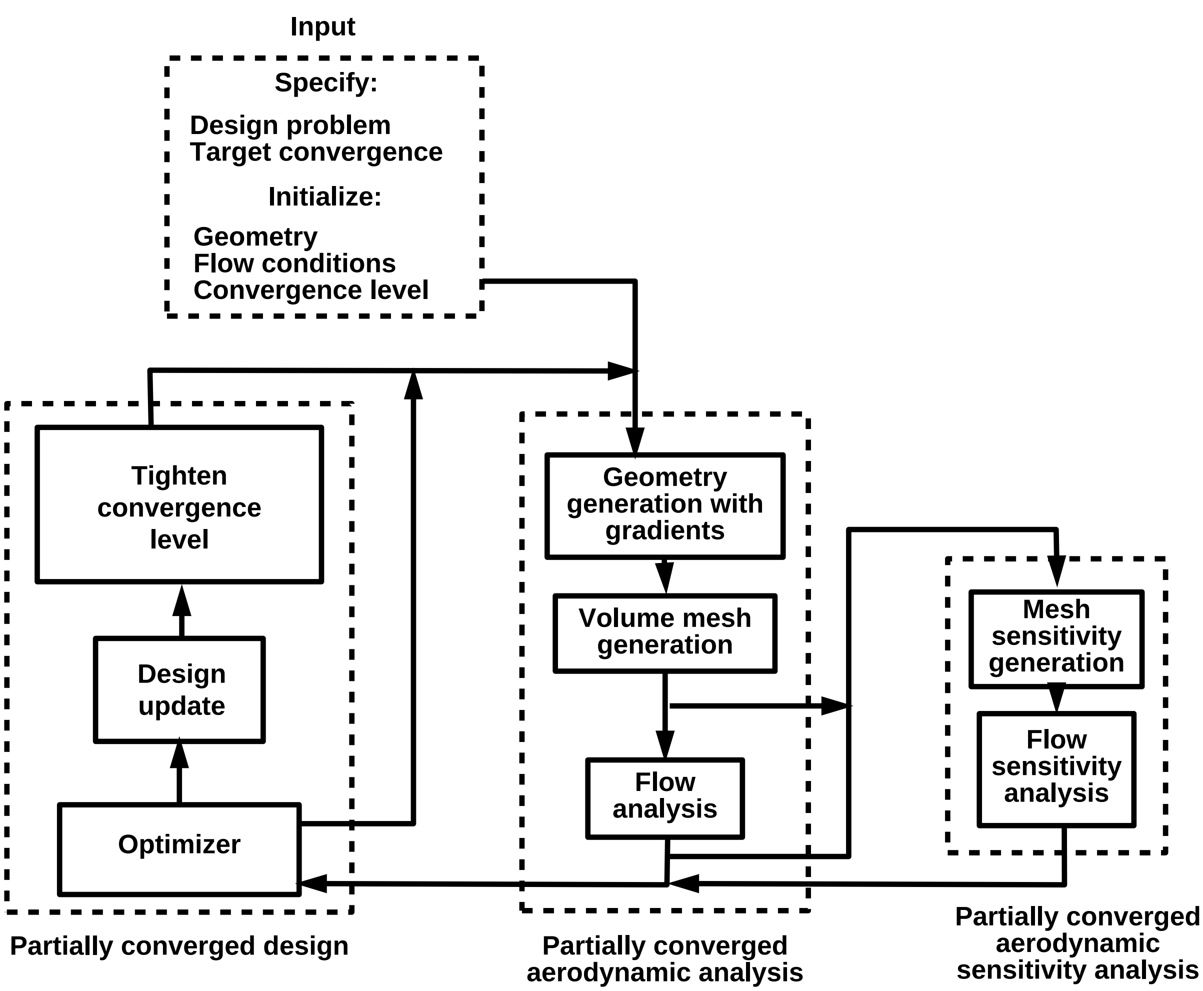

Fig. 2 Schematic diagram of the SAADO procedure.

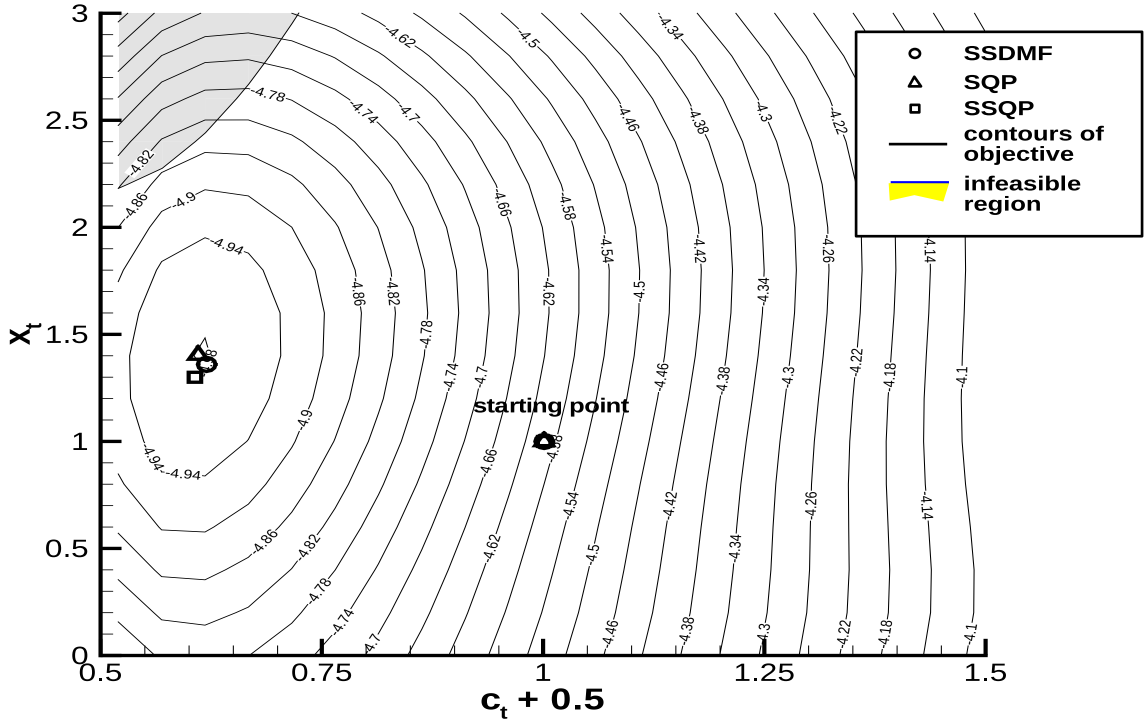

Fig. 3a Map of well−converged objective function and constraint boundary with optimization endpoints, $M_\infty = 0.5$, $\alpha = 3^\circ$.

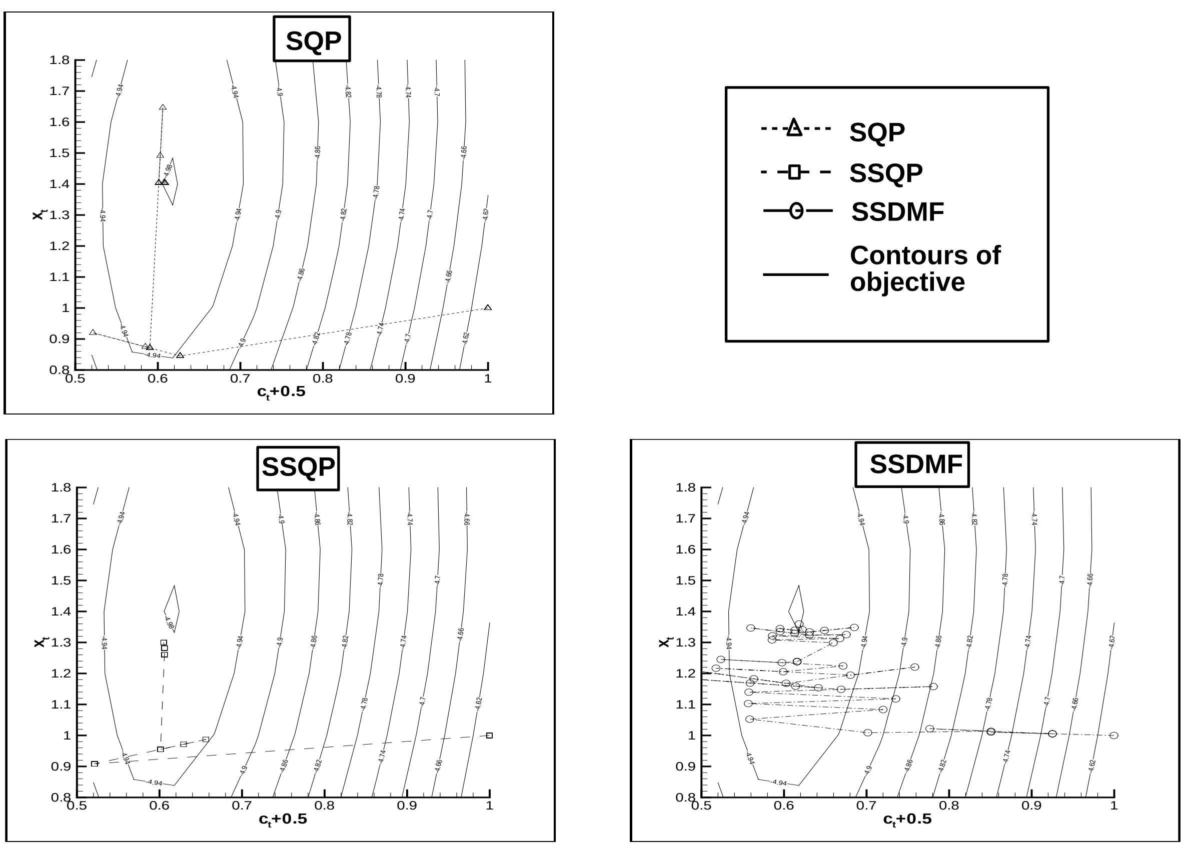

Fig. 3b. Map of well−converged objective function and constraint boundary with complete optimization paths, $M_\infty = 0.5$, $\alpha = 3^\circ$.

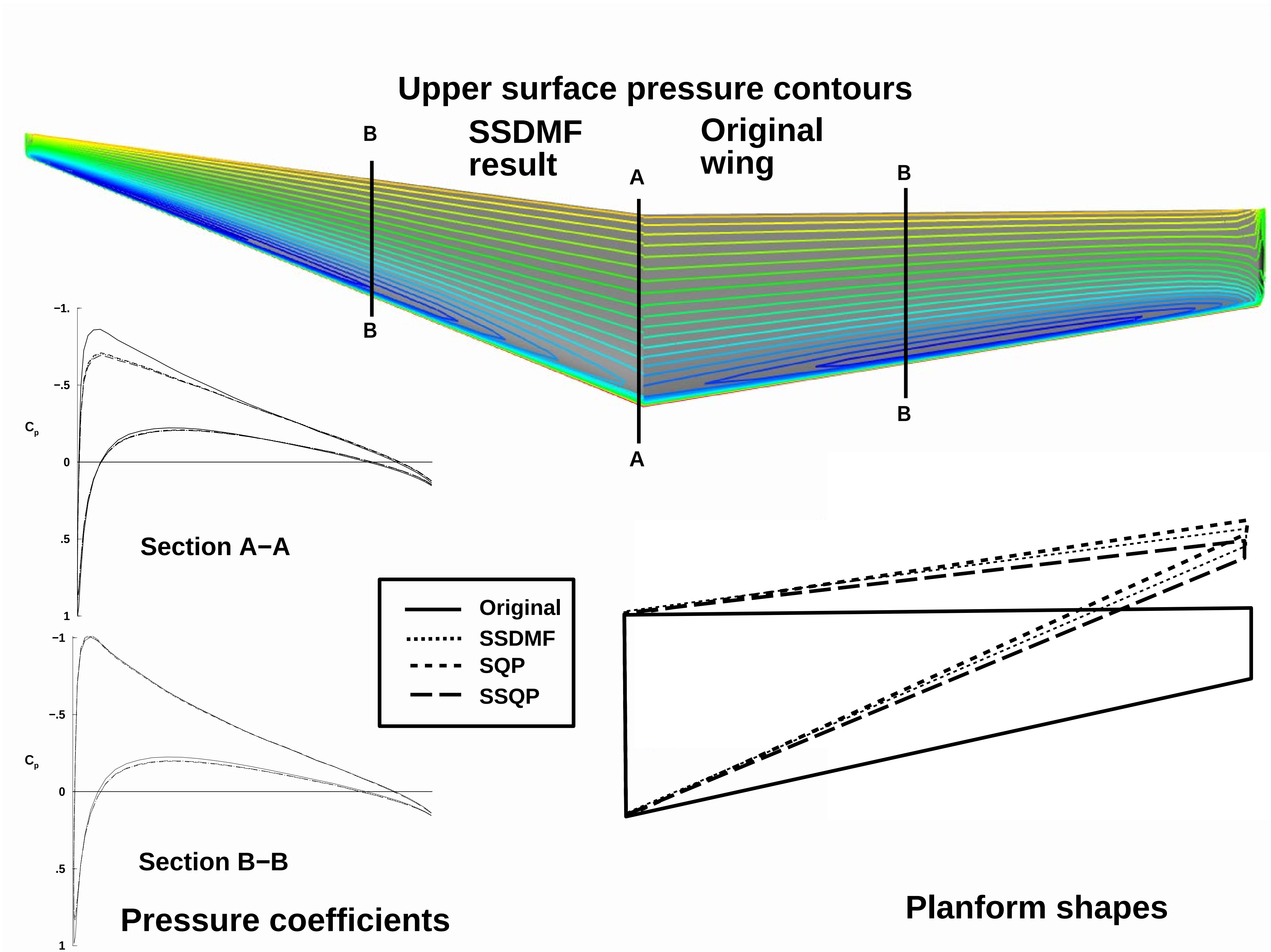

Fig. 4. Comparison of original wing and optimization results at $M_\infty = 0.5$, $\alpha = 3°$: upper surface pressure contours, pressure coefficients at two span stations, and planform shapes.

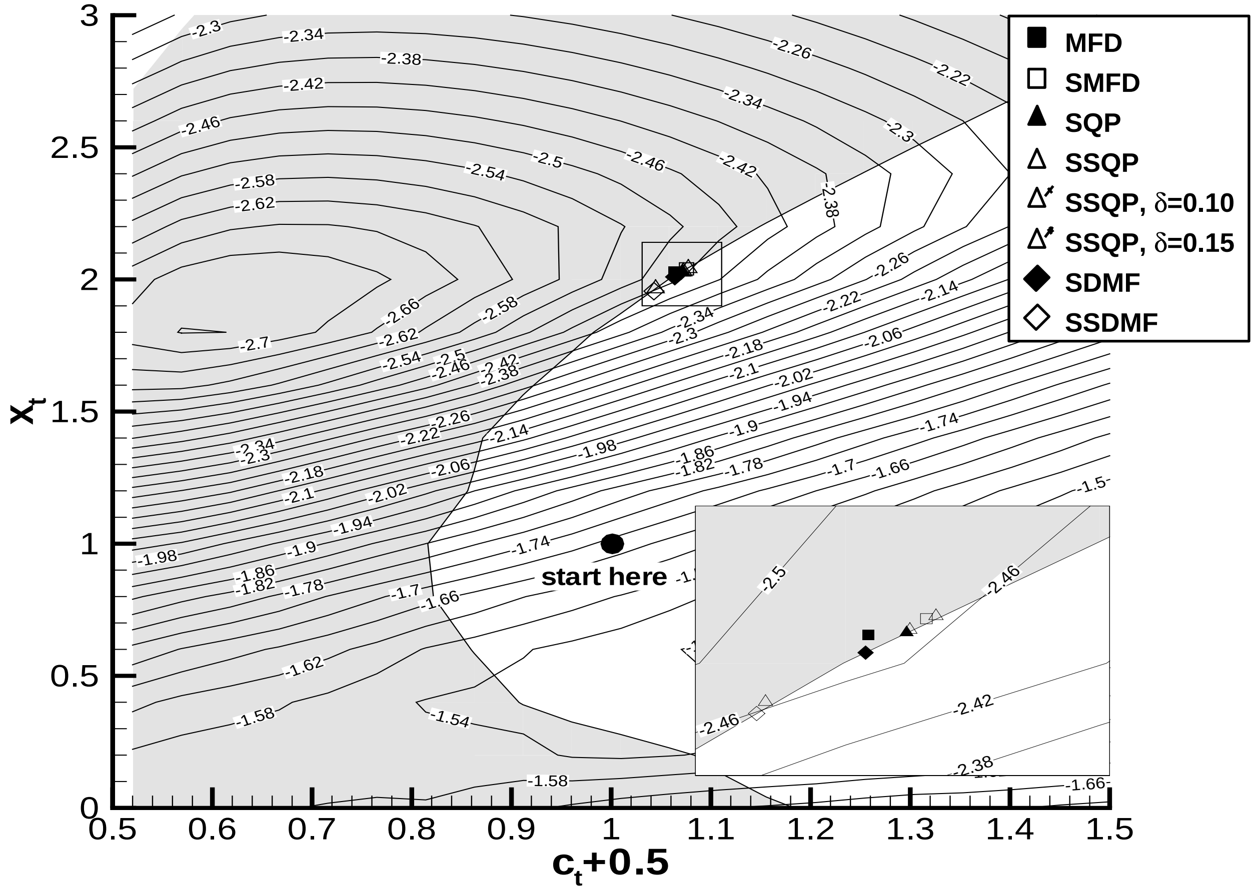

Fig. 5a Map of well−converged objective function and constraint boundary with optimization endpoints, M_∞=0.8, α=1˚.

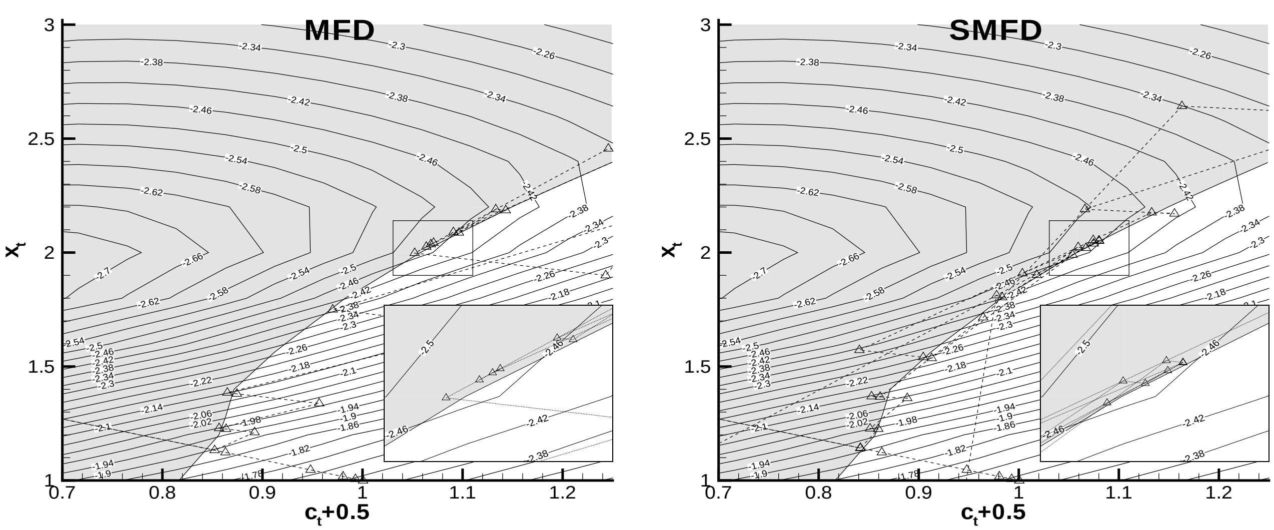

Fig. 5b Map of well−converged objective function and constraint boundary with optimization paths, M_∞=0.8, α=1˚.

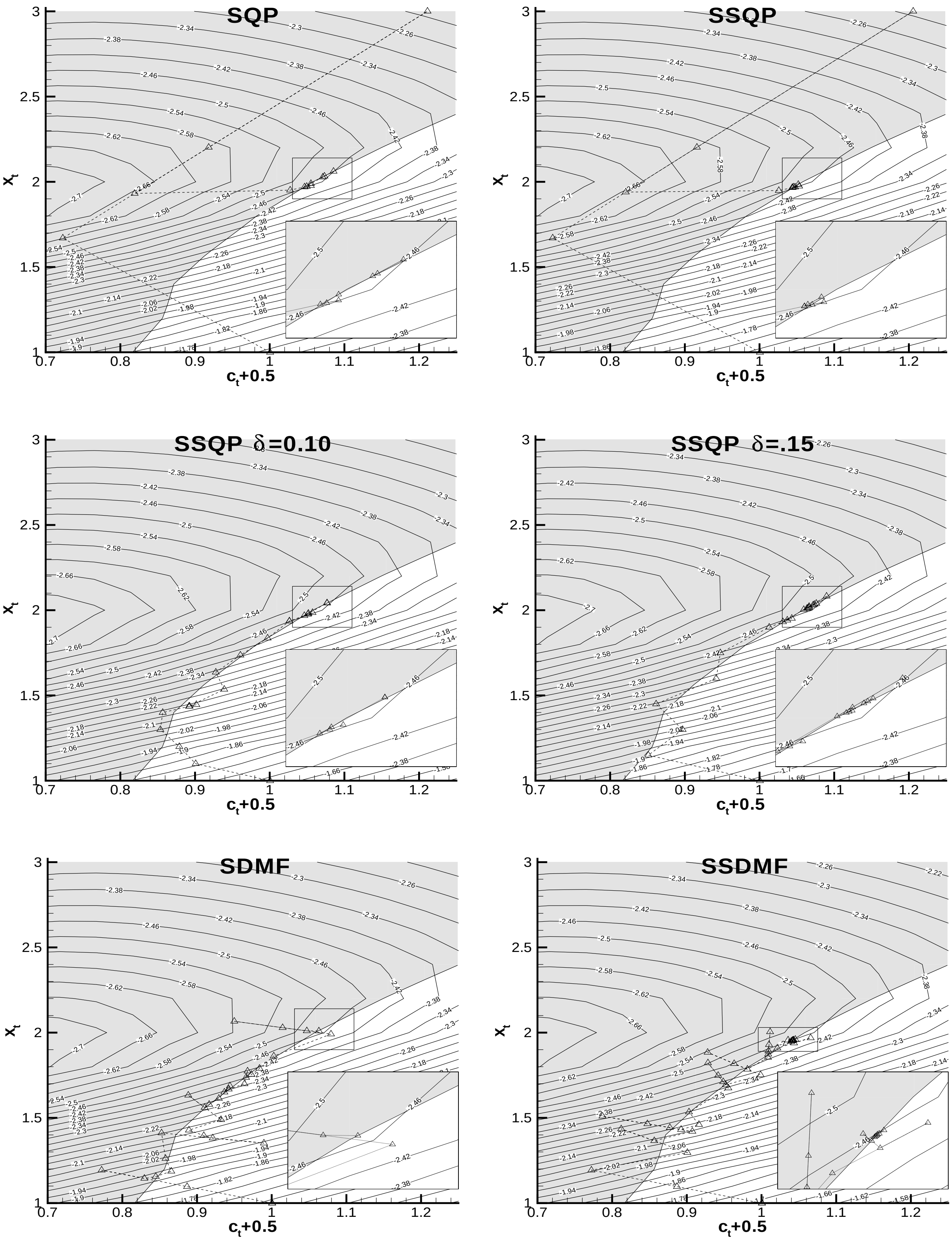

Fig. 5b Map of well−converged objective function and constraint boundary with optimization paths, M_∞=0.8, α=1˚.

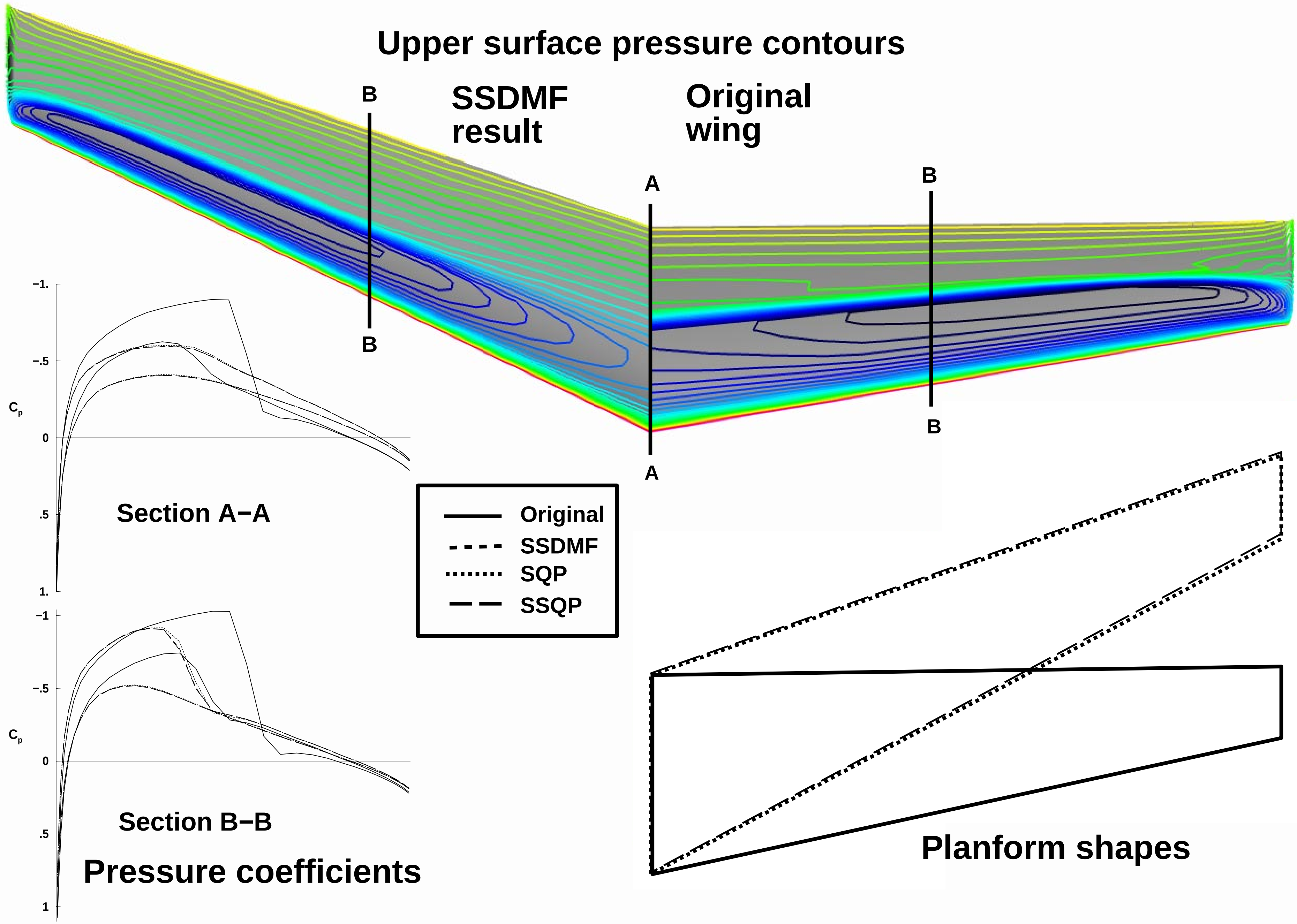

Fig. 6. Comparison of original wing and optimization results at $M_\infty = 0.8$, $\alpha = 1°$: upper surface pressure contours, pressure coefficients at two span stations, and planform shapes.

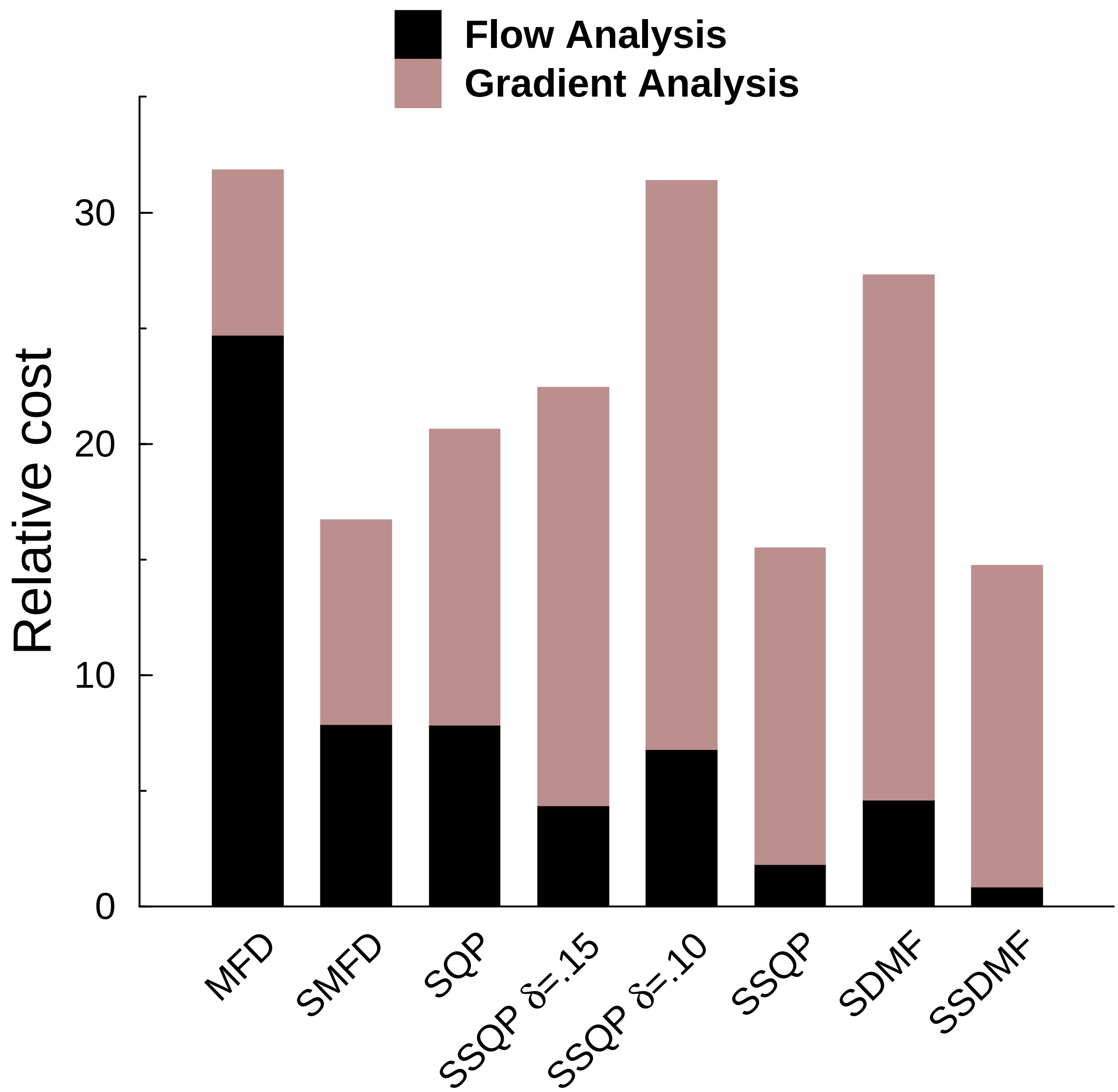

Figure 7. Computational cost comparisons